Entrance Examination Syllabus For Ph.D. (Mathematics)

(Doctor Of Philosophy In Mathematics)

Analysis: Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf.Bolzano Weierstrass theorem, Heine Boreltheorem. Continuity, uniform continuity, differentiability, mean value theorem.Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, Lebesgue measure, Lebesgueintegral.Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation. Metric spaces, compactness, connectedness. Normed Linear Spaces. Spaces of Continuous functions as examples.

Complex Analysis: Algebra of complex numbers, the complex plane, polynomials, Power series,transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem.Bilinear transform

Number Theory: Permutations, combinations, pigeon-hole principle, inclusion exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in Z, congruence's, Chinese Remainder Theorem, Euler's Ø- function, primitive roots.

Algebra: Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields.

Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations.Eigenvalues and eigenvectors, Cayley-Hamilton theorem.Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis.

Differential Equations: Existence and Uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Series solution. First and second order partial differential equations.Lagrange and Charpits methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Partial Differential Equations: Classification. Transport equation-initial value problem. Non-homogeneous equation. Laplace's equation-fundamental solution, mean value formulas, properties of harmonic functions, green's function, energy methods. Heat equation-

fundamental solution, mean value formula, properties of solutions, energy methods. Wave equation-solution by spherical means, non-homogeneous equations, energy methods. Nonlinear first order pde-complete integrals, envelopes, characteristics, hamilton-jacobi equations (calculus of variations, hamilton's ode, legendre transform). Representation of solutions-separation of variables, similarity solutions (plane and travelling waves, solitons, similarity under scaling), fourier and laplace transform, asymptotics (singular perturbations, laplace's method), power series.

Numerical Analysis: Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler and Runge-Kutta methods.

Statistics: Introduction to Statistics: Introduction to Measurement of Central Tendency and dispersion, skewness, moments and kurtosis. Probability Theory and Probability Distributions: Concepts, additive, multiplicative, conditional, probability rules, Baye'sTheorem, Binomial, Poisson and Normal distributions- their characteristics and applications Time Series: Time Series and its Components, Analysis, Models of Time Series, Methods of Studying Components of Time Series: Measurement of trend , Measurement of seasonal variations Measurement of cyclicvariations Correlation & Regression: Correlation (Karl Pearson's and Spearman's Coefficient), Methods of computing simple correlation and regression. Estimation and Testing of Hypothesis: Point and Interval Estimation, Estimator and Estimates, Confidence Intervals. Hypothesis testing and statistical influence (Introduction to methodology and Types of errors) introduction to sample tests for univariate analysis using normal distribution, t-test, z-test. Statistical Decision Theory: Decision making process, Decisions under Uncertainty and Decisions under Risk.

Operation Research: Convex sets. Linear programming problem (lpp). Examples of lpp, hyperplane, open and closed half – spaces. Feasible, basic feasible and optimal solutions, extreme point and graphical method, k-t conditions. Definition and scope of operational research, different types of models, transportation problem, replacement models and sequencing theory, inventory problems and their analytical structure. Simple deterministic and stochastic models of inventory control, basic characteristics of queuing system, different performance measures. Steady state solution of markovian queuing models: m/m/1, ww1 with limited waiting space mwc, m/m/c with limited waiting space.

Discrete Mathematics: Partially ordered sets, lattices, complete lattices, distributive lattices, complements, boolean algebra, elements of graph theory, eulerian and hamiltonian graphs, planar graphs, directed graphs, trees, spanning trees, fuzzy set theory